An SO(5) model of p-wave superconductivity and ferromagnetism

Shuichi Murakami and Naoto Nagaosa
Department of Applied Physics, University of Tokyo, Bunkyo-ku, Tokyo 113-8656, Japan

Manfred Sigrist
Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan

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We study an SO(5) model unifying p-wave superconductivity and ferromagnetism. If only a single p-wave pairing wavefunction is involved, the p-wave superconducting (pSC) and ferromagnetic (F) states can be unified to form 10-dimensional multiplet of SO(5). The collective modes in pSC and F states are studied from this viewpoint in terms of a non-linear sigma model.

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Proximity of magnetism and superconductivity is one of the most important issues in the physics of strongly correlated electronic systems. In heavy fermion systems [1,2], organic conductors [3], and high-T_c superconductors(HTSC) [4], the interplay between antiferromagnetism (AF) and anisotropic superconductivities (SC) has been studied. Recently, a theory has been proposed to unify AF and d-wave SC (dSC) in terms of the SO(5) symmetry [5]. In this theoretical framework, the order parameters of AF and dSC form a five dimensional supervector $\vec{n}$. The formation of the pseudo-gap is identified with the onset of the amplitude of this supervector $|\vec{n}|$. The SO(5) rotation in this five dimensional space turns AF into dSC and vice versa. A prediction of this theory is a specific collective mode in the superconducting state, so-called π-mode [6], which corresponds to the fluctuation towards the AF. The corresponding π-operator becomes canonically conjugate to the AF spin fluctuation below the superconducting transition and, hence, can be detected by neutron scattering experiments. This gives a possible scenario for the $41meV$-peak observed in $YBa_2CuO_{6.9}$ [7–11]. Furthermore, this unification is supported by some of the recent numerical studies [12–14].

Let us now turn to p-wave superconductors. Ferromagnetic spin fluctuations have long been known to mediate the p-wave pairing in ^{3}He [15]. A more recent example is Sr_2RuO_4 [16]. This compound shows a superconducting state below $T_c \sim 1.5$K. Unconventional pairing is clearly indicated by the high sensitivity of T_c to nonmagnetic impurities [18], and the absence of Hebel-Slichter peak in NQR spectrum [17]. Theoretically it was proposed to be a two-dimensional p-wave superconductor [20], and this was confirmed by ^{17}O NMR Knight shift measurement [19] showing the temperature-independent behavior of in-plane susceptibility. Among the five p-wave pairing states suggested from group theoretical arguments [20], $\vec{d}(\vec{k}) = \hat{z}(k_x \pm \mathrm{i}k_y)$ is the only order parameter [21] compatible with broken time-reversal symmetry observed by μSR experiments [22]. Interestingly, this compound is close to ferromagnetic order in the following aspects. First, NMR data indicate the enhancement of ferromagnetic spin fluctuations [23]. Second, high pressure destroys superconductivity and drives the material towards ferromagnetism [24]. Above 3 GPa the superconducting state has disappeared and around 8 GPa the in-plane resistivity is proportional to $T^{4/3}$ consistent with the behavior dominated by two-dimensional ferromagnetic spin fluctuations. Lastly, among related materials $Sr_{n+1}Ru_nO_{3n+1}$, ferromagnetic order is observed for $n = (2), 3, \infty$ [25–27], and with increasing n the ferromagnetic transition temperature rises.

Motivated by this, we study in this paper an SO(5) model describing the interplay between ferromagnetism (F) and p-wave superconductivity (pSC). The charge Q, the order parameter of pSC, and the total spin $\vec{S}$ constitute a ten-dimensional superspin obeying the closed SO(5) algebra. Note that this algebra was found in the context of ^{3}He superfluidity [28–30]. We consider the case where the amplitude of this superspin is developed and the rotational fluctuations between F and pSC states constitute low-lying excitations.

In the weak-coupling BCS theory one introduces a gap function, $\Delta_{\alpha\beta}(\mathbf{k}) = -\sum_{\mathbf{k}',\gamma,\delta} V_{\alpha\beta\gamma\delta}(\mathbf{k},\mathbf{k}')\langle c_{-\mathbf{k}'\gamma}c_{\mathbf{k}'\delta}\rangle$, where $V(\mathbf{k},\mathbf{k}')$ represents the electron-electron interaction. In this paper we focus on odd-parity pairing, where the gap function can be expressed as $\hat{\Delta}(\mathbf{k}) = \sum_\mu d_\mu(\mathbf{k})\mathrm{i}\sigma^\mu\sigma^y$. If we assume that $V(\mathbf{k},\mathbf{k}')$ is nonzero only when $\mathbf{k}$ and $\mathbf{k}'$ are close to the Fermi surface, $V(\mathbf{k},\mathbf{k}')$ depends only on the directions $\hat{\mathbf{k}}$ and $\hat{\mathbf{k}}'$. Hence the order parameter of spin-triplet SC state is

$$d_\mu(\hat{\mathbf{k}}) = \mathrm{i}\sum_{\mathbf{p},\alpha,\beta} V^{(t)}(\hat{\mathbf{k}},\hat{\mathbf{p}})\langle c_{-\mathbf{p}\alpha}c_{\mathbf{p}\beta}\rangle(\sigma^y\sigma^\mu)_{\alpha\beta} \qquad (1)$$

where $V^{(t)}$ is the odd-parity part of $V(\mathbf{k},\mathbf{k}')$, i.e. $V^{(t)}(\hat{\mathbf{k}},\hat{\mathbf{p}})$ is an odd function in both $\hat{\mathbf{k}}$ and $\hat{\mathbf{p}}$. The summation over $\mathbf{p}$ is restricted to the region close to the Fermi surface. The lowest order term in the expansion of $V^{(t)}(\hat{\mathbf{k}},\hat{\mathbf{p}})$ in terms of spherical harmonics yields $V^{(t)}(\hat{\mathbf{k}},\hat{\mathbf{p}}) \propto \hat{\mathbf{k}}\cdot\hat{\mathbf{p}}$, which corresponds to the pSC order parameter $(l = 1)$.

We now introduce a new operator

$$\pi_\mu(\hat{\mathbf{k}}) = \frac{1}{4}\sum_{\mathbf{p},\alpha,\beta} g(\hat{\mathbf{k}},\hat{\mathbf{p}}) c_{-\mathbf{p},\alpha}(\sigma^y\sigma^\mu)_{\alpha\beta} c_{\mathbf{p}\beta}, \qquad (2)$$

with $g(\hat{\mathbf{k}},\hat{\mathbf{p}}) \propto V^{(t)}(\hat{\mathbf{k}},\hat{\mathbf{p}})$. It obviously satisfies $\langle \pi_\mu(\hat{\mathbf{k}})\rangle \propto d_\mu(\hat{\mathbf{k}})$. The $\hat{\mathbf{k}}$-dependence indicates angular dependence of the pair wavefunction in the pSC phase. At this point we may recall that in SO(5) theory of the HTSC, the pair wavefunction is restricted to $d_{x^2-y^2}$ [5]. A similar simplification is expected, if the rotational symmetry in the orbital space is reduced to the discrete point group (tetragonal for Sr_2RuO_4). Therefore, we consider from now on a definite form of the pair wavefunction [31], which can be implemented by fixing the $\hat{\mathbf{k}}$-dependence of V, which allows us to suppress the $\hat{\mathbf{k}}$-dependence as $g(\hat{\mathbf{p}}) = g(\hat{\mathbf{k}},\hat{\mathbf{p}})$ unless necessary. Therefore, superfluid ^{3}He is outside the scope of this SO(5) theory, since the spatial part of the pair wavefunction is not fixed.

Let us define the SO(5) generators L_{ab} of rotations in the F-pSC space as elements of an antisymmetric 5×5-matrix

$$\begin{pmatrix} 0 & & & & \\ \pi_x^\dagger + \pi_x & 0 & & & \\ \pi_y^\dagger + \pi_y & -S_z & 0 & & \\ \pi_z^\dagger + \pi_z & S_y & -S_x & 0 & \\ Q & \mathrm{i}(\pi_x - \pi_x^\dagger) & \mathrm{i}(\pi_y - \pi_y^\dagger) & \mathrm{i}(\pi_z - \pi_z^\dagger) & 0 \end{pmatrix} \qquad (3)$$

where $Q = \frac{1}{2}\sum_{\mathbf{p},\alpha}\left(c^\dagger_{\mathbf{p}\alpha}c_{\mathbf{p}\alpha} - \frac{1}{2}\right)$, and $S_\mu = \frac{1}{2}\sum_{\mathbf{p},\alpha,\beta} c^\dagger_{\mathbf{p}\alpha}\sigma^\mu_{\alpha\beta}c_{\mathbf{p}\beta}$. Provided $|g(\hat{\mathbf{p}})|^2 = 1$, the generators L_{ab} satisfy SO(5) commutation rules [28–30]:

$$[L_{ab}, L_{cd}] = \mathrm{i}(\delta_{ac}L_{bd} - \delta_{ad}L_{bc} - \delta_{bc}L_{ad} + \delta_{bd}L_{ac}). \qquad (4)$$

Considering that $g(\hat{\mathbf{k}},\hat{\mathbf{p}})$ should be an odd function of $\hat{\mathbf{k}}$ and $\hat{\mathbf{p}}$, we see that $|g|^2 = 1$ exactly holds with the choice $g(\hat{\mathbf{k}},\hat{\mathbf{p}}) = \mathrm{sgn}(\hat{\mathbf{k}}\cdot\hat{\mathbf{p}})$, with fixed value of $\hat{\mathbf{k}}$. There are other choices, e.g. $g(\hat{\mathbf{p}}) = (\hat{p}_x \pm \mathrm{i}\hat{p}_y)/\sqrt{2}$, which is possibly realized in Sr_2RuO_4 [21,22].

We now define a superspin $\vec{n}$ of SO(5), which can be conveniently labeled by two indices as n_{ab}. There is some freedom to define n_{ab}, and the simplest choice is $n_{ab} = L_{ab}$. Instead, we can also define n_{ab} similarly to L_{ab}, but with an extra factor $w_{\mathbf{p}}$ [33] inside the summation over $\mathbf{p}$ in each operator, satisfying $w_{\mathbf{p}} = w_{-\mathbf{p}}$. A transformation rule of the superspin n_{ab} under SO(5) is determined by the commutation relation

$$[L_{ab}, n_{cd}] = \mathrm{i}(\delta_{ac}n_{bd} - \delta_{ad}n_{bc} - \delta_{bc}n_{ad} + \delta_{bd}n_{ac}). \qquad (5)$$

There is an obvious similarity between (4) and (5) indicating that the commutation relation (5) corresponds to the ten-dimensional adjoint representation. In contrast, in the SO(5) theory of HTSC [5], the superspin is five-dimensional, constituting the vector representation. Here we shall set $w_{\mathbf{p}} = 1$, i.e. $n_{ab} = L_{ab}$ to assure that L_{34}, L_{42}, L_{23} are the spin operators $\vec{S}$. Thus, L_{ab} plays both the roles of SO(5) generator and superspin. The physical interpretation of each element of L_{ab} is the following: S_μ and π_μ are the order parameters of the F and pSC phase, respectively, and Q is the electron number.

Now we construct a non-linear sigma model for the superspin $\vec{L}$ assuming that the length $|\vec{L}|$ has already been developed and only the direction represents low-lying degrees of freedom. This appearance of $|\vec{L}|$ occurs, in general, at a temperature higher than T_c. At the present stage it is not possible to identify clearly the onset of $|\vec{L}|$ in Sr_2RuO_4, although certain crossover behaviors have been seen in various quantities, e.g. resistance [16]. Note, however, that amplitude fluctuations of the order parameter are irrelevant in the sense of renormalization group in two dimensions. The detailed analysis of the onset of $|\vec{L}|$ is left for a future problem.

The Hamiltonian consists of the anisotropy energy in the superspin space and the elastic energy, i.e. spatial rigidity, as

$$H = \sum_{a<b}\frac{1}{2\chi_{ab}}L_{ab}^2 + \sum_{a<b}\frac{\rho_{ab}}{2}(v_{ab})^2, \qquad (6)$$

where the generalized velocity $v_{ab} = \sum_c (L_{ac}\nabla L_{bc} - L_{bc}\nabla L_{ac})$. For simplicity, we set $\rho_{ab} = \rho_{ba}$, $\chi_{ab} = \chi_{ba}$. Gauge invariance and spin rotational invariance fix the form of the first term in (6), because neither linear terms nor cross terms in L_{ab} are allowed. Since $\vec{L}$ is ten-dimensional, it might be possible to include all the rotations in the ten-dimensional space into the first term, i.e. SO(10) generators. Nevertheless, we consider here only SO(5) generators, because it is the minimal symmetry containing both SO(3) spin-rotational symmetry of F phase and U(1)-gauge symmetry of pSC. We do not know so far whether a construction of SO(10) generators in a second-quantized form is possible. It is natural to assume that χ_{ab} (ρ_{ab}) in (6) are identical with each other within the same phase. Thus, we set $\chi_{1a} = \chi_{a5} = \chi_\pi$ ($a = 2,3,4$), $\chi_{15} = \chi_Q$, $\chi_{23} = \chi_{24} = \chi_{34} = \chi_S$ and similarly for ρ. We shall also add by hand one more symmetry-breaking term

$$H_\pi = g(\vec{\pi}\times\vec{\pi}^* - \vec{\pi}^*\times\vec{\pi})^2 \qquad (g<0), \qquad (7)$$

which is proportional to $(\vec{d}\times\vec{d}^*)^2$. Since the $\vec{d}$-vector $\vec{d}(\vec{k}) = \hat{z}(k_x \pm \mathrm{i}k_y)$ realized in Sr_2RuO_4 corresponds to a unitary state, we assume $g<0$ to assign lower energy to unitary states ($\vec{\pi} \parallel \vec{\pi}^*$) in pSC than nonunitary states. This kind of term has appeared previously in Ref. [32] in connection with the discussion of the stability of the ABM state in a certain region of the phase diagram of ^{3}He. In contrast to ^{3}He, where the continuous rotational symmetry of pair wavefunctions admits five terms of this kind, here the quenched orbital part allows for only two terms: H_π and $H' = (|\mathrm{Re}\vec{d}|^2 + |\mathrm{Im}\vec{d}|^2)^2$. The latter term contributes to the anisotropy between F and pSC states.

However this anisotropy has been already taken into account in anisotropies of χ's in the first term of eq.(12) since we are considering the case with fixed $|\vec{L}|$. Therefore, H' does not give qualitatively new effects.

Let us discuss the parameters in (6). An excitation associated with the term $Q^2/(2\chi_Q)$ corresponds to a plasmon mode with high energy. Therefore, we set $\chi_Q \ll \chi_S, \chi_\pi$. The ground state is either pSC or F, depending on the values of χ_S and χ_π. If $\chi_\pi < \chi_S$ ($\chi_\pi > \chi_S$) , the system is in the F (pSC) phase.

As in [5], one can use Hamiltonian (6) (7) to study collective modes. The equation of motion is given by

$$
\begin{aligned}
\dot{L}_{ab} &= \frac{1}{2}\sum_d(\chi_{ad}^{-1}-\chi_{bd}^{-1})\{L_{ad},L_{bd}\}+\frac{1}{2}\sum_d(\rho_{ad}-\rho_{bd})\{v_{ad},v_{bd}\} \\
&+\frac{1}{2}\sum_{c,d}\rho_{ac}\nabla\{v_{ac},L_{cd}L_{bd}\}-\frac{1}{2}\sum_{c,d}\rho_{bc}\nabla\{v_{bc},L_{cd}L_{ad}\} \\
&+\frac{1}{2}\sum_{c,d}\rho_{cd}\nabla\{v_{cd},L_{ca}L_{bd}-L_{cb}L_{ad}\}+\mathrm{i}[H_\pi,L_{ab}],
\end{aligned} \quad (8)
$$

with $\{A,B\} = AB + BA$. Using (8), we can discuss collective modes in each phase: pSC and F.

When $\chi_\pi > \chi_S$ and $g < 0$, all the unitary states form a set of degenerate ground states. Without loss of generality we consider the state with $\langle L_{41}\rangle = -\langle L_{14}\rangle = A_\pi$ and other L_{ab}'s are vanishing. When we linearize the equation of motion (8) around this ground state, we get

$$\dot{L}_{c1} = (\chi_\pi^{-1}-\chi_S^{-1})A_\pi L_{c4}+\rho_\pi A_\pi^3\nabla^2 L_{c4} \quad (c=2,3) \quad (9)$$
$$\dot{L}_{c4} = -\rho_S A_\pi^3\nabla^2 L_{c1} \quad (c=2,3) \quad (10)$$
$$\dot{L}_{51} = \rho_Q A_\pi^3\nabla^2 L_{54} \quad (11)$$
$$\dot{L}_{54} = (\chi_Q^{-1}-\chi_\pi^{-1})L_{51}-\rho_\pi A_\pi^3\nabla^2 L_{51} \quad (12)$$
$$\dot{L}_{cd} = 0 \quad (c,d\in\{2,3,5\}) \quad (13)$$

At a first glance (13) seems to represent Goldstone modes, but it is not the case, because L_{cd} $(c,d\in\{2,3,5\})$ is not coupled to L_{41}. On the other hand, the rotations in spin space give rise to Goldstone modes, which are described by the coupled equations (9) and (10). The commutation relation $[\pi_\lambda,\ S_\mu] = \mathrm{i}\varepsilon_{\lambda\mu\nu}\pi_\nu$ gives canonical conjugate relation between spin and π's, when pSC order parameter in the r.h.s. is replaced by a nonzero expectation value, and $L_{24} = -S_y$ and $L_{34} = S_x$ correspond to rotations of the $\vec{d}$-vector in pSC phase, while $L_{21} = \pi_x + \pi_x^\dagger$ and $L_{31} = \pi_y + \pi_y^\dagger$ to rotations toward the F phase [32]. They yield two collective modes with the frequency

$$\omega(k) = A_\pi^2 k\sqrt{\rho_S(\chi_S^{-1}-\chi_\pi^{-1}+\rho_\pi A_\pi^2 k^2)}. \quad (14)$$

One new aspect here compared with previous discussion [32] is that the “momenta” π correspond to the fluctuation toward F state and the frequency is determined by the anisotropy energy in superspin space, $\chi_S^{-1}-\chi_\pi^{-1}$. Lastly, the mode in (11) and (12) can be neglected, because they move up to high energies when long range Coulomb interaction is considered.

When the system is in the F phase ($\chi_\pi < \chi_S$), we linearize (8) around the ground state corresponding to $\langle L_{23}\rangle = \langle S_z\rangle = A_S$ and obtain

$$\dot{L}_{c3} = (\chi_S^{-1}-\chi_\pi^{-1})A_S L_{c2}+\rho_\pi A_S^3\nabla^2 L_{c2} \quad (c=1,5) \quad (15)$$
$$\dot{L}_{c2} = (\chi_\pi^{-1}-\chi_S^{-1})A_S L_{c3}-\rho_\pi A_S^3\nabla^2 L_{c3} \quad (c=1,5) \quad (16)$$
$$\dot{L}_{43} = \rho_S A_S^3\nabla^2 L_{42} \quad (17)$$
$$\dot{L}_{42} = -\rho_S A_S^3\nabla^2 L_{43} \quad (18)$$
$$\dot{L}_{cd} = 0 \quad (c,d\in\{1,4,5\}) \quad (19)$$

(17) and (18) lead to a ferromagnetic spin-wave excitation with dispersion $\omega = \rho_S A_S^3 k^2$, while (15) and (16) couple to give two collective modes, which are fluctuations toward pSC phase. Anisotropy of χ_{ab} between pSC and F phase produces a mass in these two modes, $\omega = A_S(\chi_\pi^{-1}-\chi_S^{-1})+\rho_\pi A_S^3 k^2$. Finally, (19) is not related with collective modes, because L_{cd} $(c,d\in\{1,4,5\})$ does not couple with L_{23}.

Let us comment on the case $\chi_\pi = \chi_S$, where the symmetry is higher. At this point unitary pSC states and F states are degenerate, and the system undergoes a first-order phase transition. All the low-energy collective modes have the same dispersion $\omega = \rho_\pi A_{S,\pi}^3 k^2$.

In real materials, the spin-orbit coupling is not negligible and introduces additional anisotropy for the direction of spin and $\vec{d}$-vector. In Sr_2RuO_4, this anisotropy may fix the direction of $\vec{d}$ as $\vec{d}\parallel\hat{e}_z$ [21,22]. We treat this by adding to the Hamiltonian extra terms allowed by symmetry. The combination $k_x \pm \mathrm{i}k_y$ gives rise to a finite value of $\langle\vec{L}\rangle \parallel \hat{e}_z$, and, hence, with the spin-orbit coupling $\lambda\vec{S}\cdot\vec{L}$, we expect terms including S_z. The gauge invariance allows the following extra terms up to the second order in the order parameters.

$$H_{so} = -hS_z + h'(S_z)^2 - 2\kappa\{(\mathrm{Re}\pi_z)^2+(\mathrm{Im}\pi_z)^2\} \quad (20)$$

where κ is positive since $\langle\vec{\pi}\rangle \propto \vec{d}\parallel\hat{e}_z$. These terms alter the dispersion (14) to

$$\omega^\pm(k) = A_\pi\sqrt{(\rho_S A_\pi^2 k^2+\kappa)\left(\chi_S^{-1}-\chi_\pi^{-1}+\rho_\pi A_\pi^2 k^2+\kappa\right)}\pm h$$

i.e. the spin-orbit coupling gives a finite mass to these modes [20]. The spin susceptibility is then calculated as

$$\mathrm{Im}\chi_{xx} = \mathrm{Im}\chi_{yy} \propto \sum_{\alpha=\pm}\sum_{s=\pm1}\delta(\omega - s\omega^\alpha(k)) \quad (21)$$

On the other hand, S_z is a constant of motion. Thus, double peaks would be observable in neutron scattering, due to the spin-orbit coupling. The additional terms in H_{so} are approximate in the sense that we restrict the Hilbert space of $\vec{d}(\vec{k})$ by fixing the $\vec{k}$-dependence. Strictly speaking we have to take into account the tensorial structure due to the $\vec{k}$-dependence in the quadratic term of the

π-fields, which is introduced by the spin-orbit coupling. For the above mode they lead to a minor modification only, but leave qualitative aspects unchanged.

The above SO(5) generators and superspins can be cast into a spinor form as in [33,34], which is relevant for constructing a microscopic theory. Let us introduce a 4-component spinor

$$ {}^t\mathbf{\Psi}_{\mathbf{p}} = \left(c_{\mathbf{p}\uparrow}, c_{\mathbf{p}\downarrow}, g(\hat{\mathbf{p}})^* c^\dagger_{-\mathbf{p}\uparrow}, g(\hat{\mathbf{p}})^* c^\dagger_{-\mathbf{p}\downarrow} \right), \qquad (22) $$

where the superscript ^{t} denotes transposition. The redundancy in $\mathbf{\Psi}_{\mathbf{p}}$ is expressed as $\mathbf{\Psi}^*_{\mathbf{p}} = -g(\hat{\mathbf{p}}) R \mathbf{\Psi}_{-\mathbf{p}}$ with $R = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$. Anticommutation relations are

$$ \{\mathbf{\Psi}^\dagger_{\mathbf{p}\alpha}, \mathbf{\Psi}_{\mathbf{p}'\beta}\} = \delta_{\alpha\beta}\delta_{\mathbf{p}\mathbf{p}'},\ \{\mathbf{\Psi}^\dagger_{\mathbf{p}\alpha}, \mathbf{\Psi}^\dagger_{\mathbf{p}'\beta}\} = -g(\mathbf{p}) R_{\alpha\beta}\delta_{\mathbf{p},-\mathbf{p}'}. $$

We introduce a representation of Clifford algebra $\{\Gamma^a,\ \Gamma^b\} = 2\delta_{ab}, (a,b = 1,\cdots,5)$;

$$ \Gamma^1 = \begin{pmatrix} 0 & -\mathrm{i}\sigma^y \\ \mathrm{i}\sigma^y & 0 \end{pmatrix}, \Gamma^{(2,3,4)} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & {}^t\vec{\sigma} \end{pmatrix}, \Gamma^5 = \begin{pmatrix} 0 & \sigma^y \\ \sigma^y & 0 \end{pmatrix}. $$

in order to write SO(5) generators and superspins as

$$ L_{ab} = \frac{1}{8}\sum_{\mathbf{p}} \mathbf{\Psi}^\dagger_{\mathbf{p}} \Gamma^{ab} \mathbf{\Psi}_{\mathbf{p}}, \quad n_{ab} = \frac{1}{8}\sum_{\mathbf{p}} w_{\mathbf{p}} \mathbf{\Psi}^\dagger_{\mathbf{p}} \Gamma^{ab} \mathbf{\Psi}_{\mathbf{p}}, \quad (23) $$

where $\Gamma^{ab} = -\mathrm{i}[\Gamma^a, \Gamma^b]$. It is noteworthy that the 5-dimensional vector $\frac{1}{4}\sum_{\mathbf{p}} \mathbf{\Psi}^\dagger_{\mathbf{p}} \Gamma^a \mathbf{\Psi}_{\mathbf{p}}$, an analog of the 5-dimensional superspin [33] in the HTSC, vanishes identically. That is why the superspin in our SO(5) theory is not 5-dimensional. In the SO(5) theory of the HTSC, the situation is opposite; the 5-dimensional superspin is $\frac{1}{4}\sum_{\mathbf{p}} w_{\mathbf{p}} \Psi^\dagger_{\mathbf{p}} \Gamma^a \Psi_{\mathbf{p}+\mathbf{Q}}$ in the notation of [33], while its 10-dimensional counterpart $\frac{1}{8}\sum_{\mathbf{p}} w_{\mathbf{p}} \Psi^\dagger_{\mathbf{p}} \Gamma^{ab} \Psi_{\mathbf{p}+\mathbf{Q}}$ vanishes.

In conclusion, we have constructed an SO(5) model unifying p-wave superconductivity and ferromagnetism, which may apply to Sr_2RuO_4. This model describes a phase transition between pSC and F by varying the strength of the symmetry-breaking term. It also predicts spectra of collective modes, among which are collective modes corresponding the fluctuation between pSC and F phase. The actual phase diagram close to the transition point should be governed by these fluctuations and will be discussed elsewhere. It would be interesting if the phase transition in Sr_2RuO_4 could be observed by changing external parameters such as pressure. However, the effect of pressure is less clear in the present model, compared to SO(5) theory of HTSC, where the doping, i.e. change of chemical potential, is directly coupled to one of SO(5) generators and gives rise to a superspin-flop transition.

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[31] For a set of more than one pair wavefunction, we should deal with the relevant orbital basis functions for $\hat{\mathbf{k}}$. We consider here the two examples: (a) $\hat{\mathbf{k}} = \hat{e}_x, \hat{e}_y$, and (b) $\hat{\mathbf{k}} = \hat{e}_x, \hat{e}_y, \hat{e}_z$. In the case (a), the whole algebra decouples into a direct product of two SO(5). The generators $L^{\pm}_{ab}$ are expressed in a matrix form (3) with

$$ \pi^{\pm}_{\mu} = (1/8)\sum_{\mathbf{p},\alpha} (\mathrm{sgn}(\mathbf{p}_x) \pm \mathrm{sgn}(\mathbf{p}_y)) c_{\mathbf{p}\alpha} (\sigma^y\sigma^\mu)_{\alpha\beta} c_{-\mathbf{p}\beta}, $$
$$ Q^{\pm} = (1/4)\sum_{\mathbf{p},\alpha} (1 \pm \mathrm{sgn}(\mathbf{p}_x\mathbf{p}_y)) \left(c^\dagger_{\mathbf{p}\alpha} c_{\mathbf{p}\alpha} - 1/2 \right), $$
$$ S^{\pm}_{\mu} = (1/4)\sum_{\mathbf{p},\alpha,\beta} (1 \pm \mathrm{sgn}(\mathbf{p}_x\mathbf{p}_y)) c^\dagger_{\mathbf{p}\alpha} \sigma^\mu_{\alpha\beta} c_{\mathbf{p}\beta}. $$

The algebra decouples into two SO(5) algebras corresponding to ordering along the $(1,\pm1)$-directions. In (b),

the algebra decouples into a direct product of four SO(5) algebras, each of which corresponds to ordering along $(1,1,1)$, $(1,-1,-1)$, $(-1,1,-1)$ and $(-1,-1,1)$.

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